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Solution of the Black-Scholes equation via the Adomian decomposition method

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Abstract

The Adomian Decomposition Method (ADM) is applied to obtain a fast and reliable solution to the Black-Scholes equation with boundary condition for a European option. We cast the problem of pricing a European option with boundary conditions in terms of a diffusion partial differential equation with homogeneous boundary condition in order to apply the ADM. The analytical solution of the equations is calculated in the form of an explicit series approximation.

Keywords: *Adomian decomposition method, Black-Scholes equation, Call option, Put option.*

1 Introduction

In 1973 Fischer Black and Myron Scholes published a formula to find the price of financial options, which Robert Merton called the Black-Scholes equation [1, 2]. For their contributions, Scholes y Merton received the Nobel prize of economy, unfortunately Fisher Black passed away and could not receive it [2]. The tools used to study these types of problems are methods and ideas specialized in stochastic calculus and partial differential equations: Wilmott et al. [3], Courtadon [4] and Company *et al.* [25] used finite differences methods to approximate the solution of the option valuation equations; Geske & Johnson, MacMillan, Barone-Adesi & Whaley, Barone-Adesi & Elliot, Barone-Adesi and Whaley, and Barone-Adesi developed methods of analytic approximation [5, 6, 7, 8, 9]; Gülkaç , used a series expansion method called homotopy perturbation method to find an approximate solution for the Black-Scholes equation [10], Alawneh & Al-Khaled [28] applied the Variational Iteration Method (VIM) to solve the Fockker-Planck equation and Black-Scholes equations. Cheng *et al.* applied the homotopy analysis method [11], Bohner & Zheng [12], El-Wakil *et al.* [26] and Tatari *et al.* [27] used the Adomian decomposition method but they did not use boundary conditions to find the approximate solution of the Black-Scholes or Fockker-Planck equations.

This article presents the Adomian Decomposition Method (ADM) applied to a diffusion equation with non null Dirichlet boundary conditions obtained after reducing the Black-Scholes equation with non homogeneous boundary conditions through variable changes. The ADM, gives an analytic solution of an equation or a system of differential equations. The method is based on considering the decomposition of the unknown function in an infinite series $\sum_{n=0}^{\infty} u_n$, and the decomposition of the non linear term of the equation in another series, $\sum_{n=0}^{\infty} A_n$, where the A_n are named Adomian Polynomials. This method has its origins in the 80's when George Adomian presented and developed the then called decomposition method, to resolve linear and non linear equations, for both ordinary differential equations and partial derivative equations [13]. The method has been applied in many deterministic and stochastic problems, linear and non linear, in physics, biology, chemistry and economy [12, 14, 15, 16].

This paper is developed as follows: in section 2 we present a chage of variable that transforms the Black-Scholes equation into a diffusion differential equation. Adomian Decomposition Method is presented in section 3, and its application to Black-Scholes equation without boundary conditions and its transformation using boundary conditions is developed en section 4. Simulations of put and call options is presented en section 5. Finally, conclusions are given in section 6.

2 Put-Call options and the Black-Scholes equation.

Let us consider the problem of finding a price of an option with **maturity time** T and a cost K. The option price can be thought of as paying a prime for the right to exercise the option at maturity time. The problem is finding the "right" price of the option. To find a solution to the problem one must consider the primary characteristics of the markets, for example, randomness, one does not know how much the coin will be worth at that time [2].

Price in [2], presents the dynamic of European options: consider options on Australian dollars (AUD), purchasing and European call option on AUD with expiration time T and strike K gives the purchaser the rigth to buy one dollar at time T for a price of K dollars. Let S_T denote the price of one UAD (the exchange rate) at time T; if $S_T \leq K$, the expiration value of the option is zero; if $S_t > K$, the expiration value is $S_t - K$, since the option holder can purchase one AUD for K and immediately sell it for S_T. In the first case, the option is said to expire *out-of-the-money*, in the second case, expire *in-the-money*. Note that unless there has been some special agreement, the holder of an option that is expiring in the money does not actually have to buy the Australian dollars, but just receives the difference $S_T - K$ in cash. If P is the price paid for an option, the the final profit and loss profile is $(S_T - K)^+ - P$. An European put option with expiration T and strike K gives the right to sell one AUD at time T for K dollars, and its payoff profile is $(K - S_T)^+ - P$. If the option can be **exercised** until or at time T, it is called American option (put or call).

In financial mathematics, it can be demonstrated that by studying a strategy of self-financing one can reach the following partial differential equation called the Black-Scholes equation, formulated in 1973 by Fisher Black and Myron Scholes [1],

$$rf(t,x) = f_t(t,x) + \frac{1}{2}\sigma^2 x^2 f_{xx}(t,x) + rxf_x(t,x), \qquad x > 0,\ t \in [0,T], \tag{1}$$

where, x represents the value of the action, t the time, f the option price, r is the type of interest of the market of debt, σ is the volatility of the action, measured as the standard deviation of the logarithm of the value of the action.

In this paper we give an analytic solution of two financial options, Call Option problem,

$$\begin{cases} rC(t,x) = C_t(t,x) + \frac{1}{2}\sigma^2 x^2 C_{xx}(t,x) + rxC_x(t,x), & x > 0,\ t \in [0,T], \\ C(T,x) = max(x-K,0), & \\ C(t,x) = x - Ke^{-r(T-t)}, & when\ x \to \infty, \\ C(t,0) = 0, & \forall t > 0\ . \end{cases} \tag{2}$$

and *put option* problem,

$$\begin{cases} rP(t,x) = P_t(t,x) + \frac{1}{2}\sigma^2 x^2 P_{xx}(t,x) + rxP_x(t,x), & x > 0,\ t \in [0,T], \\ P(T,x) = max(K-x,0), & \\ P(t,x) = Ke^{-r(T-t)} - x, & when\ x \to 0, \\ P(t,x) = 0, & when\ x \to \infty,\ t \in [0,T]\,. \end{cases} \tag{3}$$

To reduce problem 2 and 3 into a diffusion problem we use change of variables given by,

$$\tau = \frac{1}{2}\sigma^2(T-t), \quad y = ln\left(\frac{x}{K}\right), \quad \gamma = \frac{2r}{\sigma^2}, \tag{4}$$

and we assume that functions $C(t,x)$ and $P(x,t)$ can be expressed by,

$$C(t,x) = Ke^{-ay-b\tau}U(\tau,y), \tag{5}$$

$$P(t,x) = Ke^{-ay-b\tau}V(\tau,y), \tag{6}$$

where $a = \frac{1}{2}\left(\frac{2r}{\sigma^2} - 1\right)$ and $b = (1+a)^2$. Thus, problem 2 transforms into

$$\begin{cases} U_t(\tau,y) = U_{xx}(\tau,y), \qquad y > 0,\ \tau \in \left[0, \frac{\sigma^2 T}{2}\right], \\ U(0,y) = max\left(e^{\frac{1}{2}(\gamma+1)y} - e^{\frac{1}{2}(\gamma-1)y}, 0\right), \\ U(\tau,L) = e^{\frac{1}{2}(\gamma+1)L + \frac{1}{4}(\gamma+1)^2\tau} - e^{\frac{1}{2}(\gamma-1)L + \frac{1}{4}(\gamma-1)^2\tau}, \\ U(\tau,0) = 0, \end{cases} \tag{7}$$

and problem 3 becomes,

$$\begin{cases} V_t\left(\tau,y\right)=V_{xx}\left(\tau,y\right), \qquad y>0,\ \tau\in\left[0,\frac{\sigma^2T}{2}\right], \\ V\left(0,y\right)=max\left(e^{\frac{1}{2}(\gamma-1)y}-e^{\frac{1}{2}(\gamma+1)y},0\right), \\ V\left(\tau,L\right)=e^{\frac{1}{2}(\gamma-1)L+\frac{1}{4}(\gamma-1)^2\tau}-e^{\frac{1}{2}(\gamma+1)L+\frac{1}{4}(\gamma+1)^2\tau}, \\ V\left(\tau,0\right)=0. \end{cases} \tag{8}$$

The general solutions of (7) and (8) are given by,

$$C\left(t,x\right)=Ke^{-\frac{1}{2}(\gamma+1)x-\frac{1}{4}(\gamma-1)^2\tau}U\left(\tau,y\right), \tag{9}$$

$$P\left(t,x\right)=Ke^{-\frac{1}{2}(\gamma-1)x-\frac{1}{4}(\gamma+1)^2\tau}V\left(\tau,y\right). \tag{10}$$

Thus, to obtain a solution for put and call option problems of the form (9) and (10) we reduce equations (2) and (3) into equations (7) and (8), that is, we have reduced the Black-Scholes equation into a diffusion equation in order to use all given boundary conditions.

3 The Adomian Decomposition Method (ADM)

The Adomian decomposition method allows us to find an analytic solution in the form of the series [2, 16] and consists in identify the linear and non linear parts of the equation in order to integrate the highest order differential operator in the linear part, and then consider the unknown function as a series that has well determined components. Then, the non linear function is decomposed into Adomian polynomial terms. We define the initial and boundary conditions and the independent function as an initial approximation, and the terms of the solution series are found in a successive fashion by a recurrence relation.

Given a differential equation,

$$Fu(t)=g(t), \tag{11}$$

where F represents a non linear differential operator which includes both linear and non linear terms, so that equation 11 can be written as

$$Lu\left(t\right)+Ru\left(t\right)+Nu(t)=g(t), \tag{12}$$

where $L+R$ is the linear operator, L is an easily invertible operator, R the remainder linear operator, N represents the non linear operator and g is the independent function of $u\left(t\right)$.

Resolving for $Lu\left(t\right)$,

$$Lu\left(t\right)=g\left(t\right)-Ru\left(t\right)-Nu(t).$$

Since L is invertible, we have that,

$$L^{-1}Lu\left(t\right)=L^{-1}g\left(t\right)-L^{-1}Ru\left(t\right)-L^{-1}Nu(t).$$

An equivalent expression

$$u\left(t\right)=\varphi+L^{-1}g\left(t\right)-L^{-1}Ru\left(t\right)-L^{-1}Nu(t), \tag{13}$$

where φ is the integration constant and satisfies $L\varphi=0$. For problems with an initial value in $t=a$, we have conveniently defined L^{-1} for $L=\frac{d^ny}{dx^n}$, which is the definite integrate of a to t.

This method assumes a solution in the form of an infinite series for the unknown function $u\left(t\right)$ given by,

$$u\left(t\right)=\sum_{i=0}^{\infty}u_i(t). \tag{14}$$

The non linear term $Nu(t)$ is decomposed as

$$Nu\left(t\right)=\sum_{n=0}^{\infty}A_n(u_0,u_1,\ldots,u_n), \tag{15}$$

where A_n is called and Adomian polynomial, and depends on the particularity of the non linear operator. The A_n's are calculated in a general way by the following formula:

$$A_n(u_0,u_1,\ldots,u_n)=\frac{1}{n!}\frac{d^n}{d\lambda^n}N\left(\sum_{j=0}^{\infty}\lambda^j u_j\right)\Bigg|_{\lambda=0}. \tag{16}$$

Equation 16 can be solved using a software, such as MATLAB or MAPLE [17].
Substituting 14 and 15 in the equation 13 we have,

$$\sum_{i=0}^{\infty}u_i(t)=\varphi+L^{-1}g\left(t\right)-L^{-1}R\sum_{i=0}^{\infty}u_i(t)-L^{-1}\sum_{n=0}^{\infty}A_n(u_0,u_1,\ldots,u_n).$$

And thus a solution is obtained by

$$\begin{cases} u_0\left(t\right)=\varphi+L^{-1}g \\ u_{n+1}\left(t\right)=-L^{-1}Ru_n(t)-L^{-1}A_n(u_0,u_1,\ldots,u_n). \end{cases} \tag{17}$$

The approximations are given by

$$\psi_k=\sum_{i=0}^{k-1}u_i(t). \tag{18}$$

The decomposition of the solution series converges in general very quickly. This means thaw few terms are required for the approximation. Convergence of this method has been rigorously established by Cherruault [18], Cherruault and Adomian [19], and Abbaoui and Cherruault [20, 21].

4 Solutions of the Black-Scholes equation through ADM.

4.1 ADM direct application.

Let us consider equation 1 with terminal function $f\left(T,x\right)=f_T$, where, $f_T=max\left(x-K,0\right)$, for a call option, or $f_T=max\left(K-x,0\right)$, for a put option. ADM can be applied using operators as shown in [12],

$$L=\left(.\right)_t,\ R=\tfrac{1}{2}\sigma^2x^2(.)_{xx}+rx(.)_x-r\left(.\right),\ N=0,\text{ y } g=0.$$

Rewriting equation 1 we have that,

$$f_t\left(t,x\right)=-\frac{1}{2}\sigma^2x^2f_{xx}\left(t,x\right)-rxf_x\left(t,x\right)+rf\left(t,x\right),$$

applying $L^{-1}=\int_t^T(.)ds$ in both sides of the equation, we obtain,

$$L^{-1}f_t\left(t,x\right)=-\frac{1}{2}\sigma^2L^{-1}x^2f_{xx}\left(t,x\right)-rL^{-1}xf_x\left(t,x\right)+rL^{-1}f\left(t,x\right),$$

$$f\left(T,x\right)-f\left(t,x\right)=-\frac{1}{2}\sigma^2\int_t^T x^2f_{xx}\left(s,x\right)ds-r\int_t^T xf_x\left(s,x\right)ds+r\int_t^T f\left(s,x\right)ds,$$

$$f\left(t,x\right)=f_T+\frac{1}{2}\sigma^2\int_t^T x^2f_{xx}\left(s,x\right)ds+r\int_t^T xf_x\left(s,x\right)ds-r\int_t^T f\left(s,x\right)ds.$$

Assuming that the solution could be expressed in terms of an infinite series,

$$f\left(t,x\right)=\sum_{i=0}^{\infty}f_i(t,x),$$

We obtain that,

$$\sum_{i=0}^{\infty} f_i(t,x) = f_T + \frac{1}{2}\sigma^2 \int_t^T x^2 \sum_{i=0}^{\infty} f_{ixx}(s,x)ds + r\int_t^T x \sum_{i=0}^{\infty} f_{ix}(s,x)ds - r\int_t^T \sum_{i=0}^{\infty} f_i(s,x)ds,$$

$$\sum_{i=0}^{k} f_i\left(t,x\right) = f_T + \frac{1}{2}\sigma^2 \sum_{i=0}^{k} \int_t^T x^2 f_{ixx}\left(s,x\right)ds + r\sum_{i=0}^{k} \int_t^T x f_{ix}\left(s,x\right)ds - r\sum_{i=0}^{k} \int_t^T f_i\left(s,x\right)ds.$$

Each term of the approximation is represented by

$$\begin{cases} f_0\left(t,x\right) = f_T, \\ f_{n+1}(t,x) = \frac{1}{2}\sigma^2 \int_t^T x^2 f_{nxx}(t,x)ds + r\int_t^T x f_{nx}(t,x)ds - r\int_t^T f_n(t,x)ds, \quad for \quad n > 0. \end{cases} \tag{19}$$

Then, an approximation is given by the partial sum

$$f\left(t,x\right) \approx \psi_{k+1} = \sum_{i=0}^{k} f_i(t,x). \tag{20}$$

Observe that the boundary conditions were not used, when there is a boundary value problem, a special treatment will be executed as shown in section 4.3.

4.2 ADM application for european options.

Given the system

$$\begin{cases} u_t\left(\tau,y\right) = u_{xx}\left(\tau,y\right), \qquad y > 0, \; \tau \in \left[0, \frac{\sigma^2 T}{2}\right], \\ u\left(0,y\right) = u_0\left(y\right). \end{cases} \tag{21}$$

Following the AMD algorithm, considering $L = \frac{du}{d\tau}$, $R = \frac{d^2u}{dx^2}$, $N = 0$, y $g = 0$, we obtain,

$$\begin{aligned} L^{-1}u_t\left(\tau,y\right) &= L^{-1}u_{xx}\left(\tau,y\right), \\ u\left(\tau,y\right) &= u\left(0,y\right) + \int_0^{\tau} u_{xx}\left(s,y\right)ds. \end{aligned}$$

Assuming a solution in the form of a infinite series $u\left(\tau,y\right) = \sum_{i=0}^{\infty} u_i\left(\tau,y\right)$, we have,

$$\sum_{i=0}^{\infty} u_i\left(\tau,y\right) = u\left(0,y\right) + \int_0^{\tau} \sum_{i=0}^{\infty} u_{ixx}\left(s,y\right)ds.$$

For an approximation up to $k+1$ terms, we have,

$$\begin{aligned} &\sum_{i=0}^{k} u_i\left(\tau,y\right) = u\left(0,y\right) + \int_0^{\tau} \sum_{i=0}^{k} u_{ixx}\left(s,y\right)ds, \\ \Leftrightarrow &\sum_{i=0}^{k} u_i\left(\tau,y\right) = u\left(0,y\right) + \sum_{i=0}^{k} \int_0^{\tau} u_{ixx}\left(s,y\right)ds. \end{aligned}$$

Thus, the $(k+1)$-th approximation for the solution is given by,

$$\psi_k = \sum_{i=0}^{k-1} u_i(t) \approx u\left(t\right). \tag{22}$$

And so, the solution for the original problem is determined for the call option by

$$C\left(t,x\right) = Ke^{-\frac{1}{2}(\gamma+1)x - \frac{1}{4}(\gamma-1)^2\tau}\psi_k, \tag{23}$$

and for the put option,

$$P\left(t,x\right) = Ke^{-\frac{1}{2}(\gamma-1)x - \frac{1}{4}(\gamma+1)^2\tau}\psi_k. \tag{24}$$

4.3 Solution of the diffusion equation with boundary conditions via the ADM

Adomian Decomposition method is not appropriate for resolving partial differential equations with non homogeneous boundary conditions, however, under a change of a variable, the initial value and non homogeneous boundary conditions problem can be transformed into one of initial value with homogeneous boundary as mentioned in [23, 24]. Transforming the original problem following the methodology presented before by Luo et al. [24], assume that

$$U(\tau, y) = u(\tau, y) + w(\tau, y),$$

where,

$$w(\tau, y) = U(0, y) + (U(0, y) - U(\tau, L))\left(\frac{y - y0}{L - y0}\right),$$

and so, as problems 7 and 8 are similar can be written in a general form as follows,

$$\begin{cases} u_t(\tau, y) = u_{xx}(\tau, y) - w_t(\tau, y), \quad y > 0, \ \tau \in \left[0, \frac{\sigma^2 T}{2}\right], \\ u(0, y) = u_0(y) - w(0, y), \\ u(\tau, L) = 0, \\ u(\tau, 0) = 0, \qquad \forall \tau > 0. \end{cases} \tag{25}$$

Where, if we have a call option

$$u_0(y) = \max\left(e^{\frac{1}{2}(\gamma+1)y} - e^{\frac{1}{2}(\gamma-1)y}, 0\right),$$

or, if the problem corresponds to a put option

$$u_0(y) = \max\left(e^{\frac{1}{2}(\gamma-1)y} - e^{\frac{1}{2}(\gamma+1)y}, 0\right).$$

Now, following the ADM algorithm, let us consider $L = \frac{du}{d\tau}$, $R = \frac{d^2u}{dx^2}$, $N = 0$ y $g = -w_t(\tau, y)$, and so,

$$\begin{aligned} L^{-1}u_t(\tau, y) &= -L^{-1}w_t(\tau, y) + L^{-1}u_{xx}(\tau, y), \\ u(\tau, y) &= u(0, y) - L^{-1}w_t(\tau, y) + \int_0^\tau u_{xx}(s, y)\, ds. \end{aligned}$$

Considering a solution in the form of the infinite series $u(\tau, y) = \sum_{i=0}^{\infty} u_i(\tau, y)$, we have,

$$\sum_{i=0}^{\infty} u_i(\tau, y) = u(0, y) - L^{-1}w_t(\tau, y) + \int_0^\tau \sum_{i=0}^{\infty} u_{ixx}(s, y) ds.$$

For an approximation up to $k + 1$ terms,

$$\begin{aligned} &\textstyle\sum_{i=0}^{k} u_i(\tau, y) = u(0, y) - L^{-1}w_t(\tau, y) + \int_0^\tau \sum_{i=0}^{k} u_{ixx}(s, y) ds, \\ \Leftrightarrow &\textstyle\sum_{i=0}^{k} u_i(\tau, y) = u(0, y) - L^{-1}w_t(\tau, y) + \sum_{i=0}^{k} \int_0^\tau u_{ixx}(s, y)\, ds. \end{aligned}$$

The terms of the series are completely determined by

$$\begin{cases} u_0(t) = u(0, y) - L^{-1}w_t(\tau, y), \\ u_{n+1}(t) = \int_0^\tau u_{ixx}(s, y)\, ds. \end{cases} \tag{26}$$

Thus, the $(k + 1)$-th approximation for the solution is given by

$$u(t) \approx \psi_k = \sum_{i=0}^{k-1} u_i(t). \tag{27}$$

The solution for the original problem is determined for the call option by

$$C(t, x) = K e^{-\frac{1}{2}(\gamma+1)x - \frac{1}{4}(\gamma-1)^2\tau}(\psi_k + w(\tau, y)), \tag{28}$$

and for the put option

$$P(t, x) = K e^{-\frac{1}{2}(\gamma-1)x - \frac{1}{4}(\gamma+1)^2\tau}(\psi_k + w(\tau, y)). \tag{29}$$

5 Simulations

In this section we compare the results of the traditional ADM and methodology presented in this article applied to problems (2) and (3), sections 4.2 and 4.3. In the first case, ADM is applied solely using the initial condition, just as it was done in Bohner [12], in the second case ADM was applied using the methodology presented in 4.3, transforming the Black-Scholes problem into one of diffusion, and with that we could use the boundary conditions of (2) and (3).

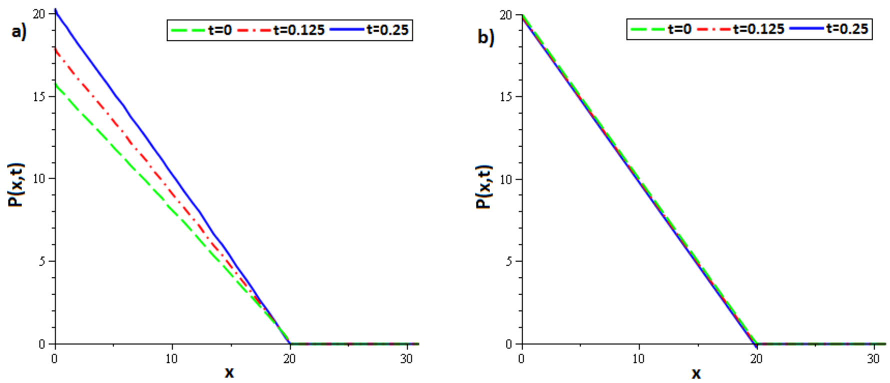

Figure 1: Solution approximation: a) with boundary conditions, b) without boundary conditions.

Let us define the group of parameters $r = 0.05, \sigma = 0.317, K = 20$ and $T = 0.25$ (3 months) for the problems (2) and (3). Simulations shows solution time profiles at $t = 0, 0.125$ and 0.25 (start, 1.5 months and maturity time T).

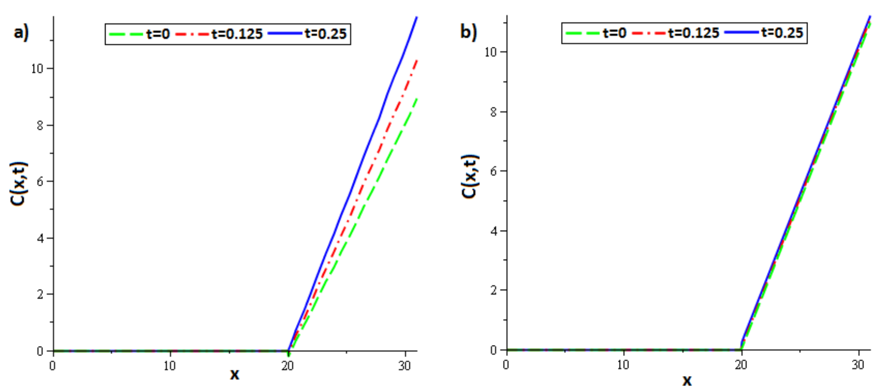

Figure 2: Solution approximation: a) with boundary conditions, b) without boundary conditions.

In Fig. 1a) we show the solution approximation for call option problem (2) using formula (27) with $k = 10$, applying the ADM on homogeneous dirichlet boundary diffusion problem (24) obtained from problem (7), and so, in Fig. 1b), we presented the solution approximation profiles obtained from problem (2) applying the ADM only using the initial condition (see section 4.1 and [12]).

Analogously, in Fig. 2 one can to observe a similar behavior in profiles at $t = 0, 0.125$ and 0.25 for put option solution approximation. In Fig. 2a) an approximation solution to put option problem (3) is presented, using the methodology developed in 4.3 with $k = 10$, in which we applied the ADM on problem (25), and then using equality (29). In Fig. 2b) we can see the result of applying ADM without considering boundary conditions.

In both cases, call and put option problems, one can observe differences in profiles obtained from approximations by using all boundary conditions and approximation using only the initial function in the ADM, this differences causing possible money losses in real applications.

6 Conclusions

Since Adomian descomposition method converges quickly as shown by Cherruault [18], Adomian and Cherruault [19], Abbaoui y Cherruault [20, 21], it turns out to be an efficient alternative tool to solve the Black-Scholes equation problem, in general, ADM gives an analytic solution for partial differential equation problems, without implying that this solution is adequate to a given problem, because it does not use all boundary conditions, however, we showed that ADM is applicable to partial differential equations with null Dirichlet boundary conditions transforming it into a diffusion equation with null Dirichlet conditions.Simulations shows the efficiency of the method. Therefore, the methodology presented in this article may be very useful for practitioners.

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References

[1] F. Black, M. Scholes, The pricing of option and corporate liabilities, The Journal of Political Economy, 81, (1973), 637–654.

[2] J.F. Price, Optional mathematics is not optional, Notices of the American Mathematical Society, 43, (1996), 964-971.

[3] P. Wilmott, S. Howison, J. Dewynne, The Mathematics of Financial Derivatives, Cambridge University Press, Cambridge, (1997).

[4] G. Courtadon, A more accurate finite difference approximations for the valuation of options, Journal of Financial and Quantitative Analysis, 17, (1982), 697–703.

[5] R. Geske, H. Johnson, The American put options valued analytically, The Journal of Finance, 39, (1984), 1511–1524.

[6] L.W. MacMillan, An analytical approximation for the American put option, Advances in Futures and Options Research, 1, (1986), 119–139.

[7] G. Barone-Adesi, R Whaley, Efficient analytic approximation of American option values, The Journal of Finance, 42, (1978), 301–320.

[8] G. Barone-Adesi, R. Elliot, Approximations for the values of American options, Stochastic Analysis and Applications, 9(2), (1991), 115–131.

[9] G. Barone-Adesi, The saga of the American put. Journal of Banking and Finance, 29(11), (2005), 2909–2918.

[10] V. Gülkaç, The homotopy perturbation method for the Black-Scholes equation, Journal of Statistical Computation and Simulation, 80(12), 1349-1354.

[11] J. Cheng, S.-P. Zhu, S.-J. Liao, An explicit series approximation to the optimal exercise boundary of American put options, Communications in Nonlinear Science and Numerical Simulations, 15(5), (2010), 1148-1158.

[12] M. Bohner, Y. Zheng, On analytical solutions of the Black-Scholes equation, Applied Mathematics Letters, 22(3), (2009), 309-313.

[13] G. Adomian, R. Rach, Nonlinear stochastic differential delay equations. Journal of Mathematical Analysis and Applications, 91(1), (1983), 94-101.

[14] M. Pujol, P. Grimalt, A non-linear model of cerebral diffusion: stability of finite differences method and resolution using the Adomian method, International Journal of Numerical Methods for Heat and Fluid Flow, 13(4), (2003), 473-485.

[15] O. D. Makinde, Solving ratio-dependent predator-prey system with constant effort harvesting using adomian decomposition method, Applied Mathematics and Computation, 186(1), (2007), 17-22.

[16] L. Blanco-Cocom, E. Ávila-Vales, The use of the Adomian decomposition method for a Sirc influenza model, Advances in Differential Equations and Control Processes, 5(2), (2010), 115-127.

[17] W. Chen and Z. Lu, An algorithm for Adomian Decomposition Method, Applied Mathematics and Computation, 159(1), (2004), 221-235.

[18] Y. Cherruault, Convergence of Adomian's method, Kybernetes, 18(2) (1989), 31-38.

[19] Y. Cherruault, G. Adomian, Decomposition methods: a new proof of convergence, Mathematical and Computer Modelling, 18(12), (1993), 103-106.

[20] K. Abbaoui, Y. Cherruault, Convergence of Adomian's method applied to differential equations, Computers and Mathematics with Applications, 28(5), (1994), 103-109.

[21] K. Abbaoui, Y. Cherruault, New ideas for proving convergence of decomposition methods, Computers and Mathematics with Applications, 29(7), (1995), 103-108.

[22] S. Shuka, Finite-difference methods for pricing the American put option, Advanced Mathemathics if finance Honours Project (2001), URL: http://www.mendeley.com/research/advanced-mathematics-finance-honours-project-finitedifference-methods-pricing-american-put-option/

[23] G. Adomian, R. Rach. A further consideration of partial solutions in the decomposition method, Computers and Mathematics with Applications, 23(1), (1992), 51–64.

[24] X.-G. Luo, Q.-B. Wu, B.-Q. Zhang, Revisit on partial solutions in the Adomian decomposition method: Solving heat and wave equations, Journal of Mathematical Analysis and Applications, 321(1), (2006), 353-363

[25] R. Company, L. Jódar, J.-R. Pintos, A numerical method for European Option Pricig with transactions costs nonlinear equation, Mathematical and Computer Modelling, 50(5-6), (2009), 910–920.

[26] S.A. El-Wakil, M.A. Abdou, A. Elhanbaly, Adomian decomposition method for solving the diffusion-convection-reaction equations, Applied Mathematics and Computation, 177, (2006), 727–736.

[27] M. Tatari, M. Dehghan, M. Razzaghi, Application of the Adomian decomposition method for the Fokker-Plank equation, Mathematical and Computer Modelling, 45(5-6), (2007), 639–650.

[28] A. Alawneh, K. Al-Khaled, Numerical treatment of stochastic models used in statistical system and financial markets. Journal Computers and Mathematics with Applications, 56(10), (2008), 2724–2732.